A MODEL OF A HARDENING PLASTIC MATERIAL

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An attempt is made in [1] to construct a phenomenological model of a hardening elastoplastic body. The following notation are introduced: Σ, Σ_Δ, $\Omega_{p\Delta}$ are the stress tensors, stress increments, and plastic strain increments, σ_1, σ_2, $\sigma_3(\sigma_1 \geq \sigma_2 \geq \sigma_3)$ and Δe_1, Δe_2, Δe_3 are the principal values of the tensors Σ and $\Omega_{p\Delta}$. The following fundamental assumption II is taken among the 11 explicitly formulated hypotheses: if the direction of the principal axes Σ_Δ agrees with the direction of the principal axes Σ, then the direction of the principal axes $\Omega_{p\Delta}$ also agrees with the direction of the principal axes Σ. If the loading Σ_Δ does not alter the principal axes Σ, then the plastic strains are found by using the condition of plastic incompressibility and the differential realtionships (assumption IV):

$$\Delta\gamma_{13} = \frac{\Delta T_{13}}{g_{13}} + \frac{\Delta T_{12}}{2g_{12}}(1+\lambda_{12}) + \frac{\Delta T_{23}}{2g_{23}}(1+\lambda_{23}),$$

$$\Delta\gamma_{12} = \frac{\Delta T_{13}}{2g_{13}}(1+\lambda_{13}) + \frac{\Delta T_{12}}{g_{12}} - \frac{\Delta T_{23}}{2g_{23}}(1-\lambda_{23}),$$

$$\Delta\gamma_{23} = \frac{\Delta T_{13}}{2g_{13}}(1-\lambda_{13}) - \frac{\Delta T_{12}}{2g_{12}}(1-\lambda_{12}) + \frac{\Delta T_{23}}{g_{23}},$$

$$2T_{13} = \sigma_1 - \sigma_3,\ 2T_{12} = \sigma_1 - \sigma_2,\ 2T_{23} = \sigma_2 - \sigma_3;\ \Delta\gamma_{ij} = \Delta e_i - \Delta e_j.$$

The properties of the material are given here by the moduli g_{ij}, and the magnitudes of the parameters λ_{ij} are found taking account of the boundary conditions by using assumption VIII, the principle of maximality of the plastic work increments (the maximum is found by means of λ_{ij} under the additional conditions $|\lambda_{ij}| \leq 1$). The magnitudes of the moduli g_{ij} should be determined from tests, they are distinct in the complete ($T \geq \tau_s$, $T_1 \geq \tau_{s1}$) and incomplete ($T \geq \tau_s$, $T_1 < \tau_{s1}$) plasticity states (the quantities T_{ij} are denoted in terms of T, T_1, T_2 in decreasing order, and τ_s and τ_{s1} are characteristics of the material which have been found experimentally). The magnitudes of the moduli g_{ij} also depend on the character of the loading: $\Delta T > 0$, $\Delta T_1 > 0$, $\Delta T_2 > 0$ is active loading: $\Delta T < 0$, $\Delta T_1 < 0$, $\Delta T_2 < 0$ is the general unloading $\Omega_{p\Delta} = 0$ (assumption III), the remaining kinds of loading (corresponding to other combinations of ΔT, ΔT_1, ΔT_2) are distinct kinds of partial unloadings. The plastic strain continues with partial unloading in the model being described, where the magnitudes of the moduli g_{ij} change as compared with their values during active loading; for corresponding constraints on ΔT, ΔT_1, ΔT_2 partial hardening sets in, for which some of the g_{ij} becomes infinite. The magnitudes of the g_{ij} vary when going from loading of one type over to loading of another type.

The loading which alters the direction of the principal axes of Σ, is represented as the sum of a quasisimple (without altering the principal axes of Σ) and an orthogonal (without altering the principal values of Σ) loading, their corresponding $\Omega_{p\Delta}$ are summed (assumption X). Relationships (assumption XI) between the components $\Omega_{p\Delta}$ and Σ_Δ, which are analogous to those proposed earlier in [2], are taken for the orthogonal loading. These relationships contain the additional material characteristics which are used to find the position of the principal axes of $\Omega_{p\Delta}$ relative to the principal axes of Σ, and the principal values $\Omega_{p\Delta}$.

The quantities g_{ij} were determined from part of the data of four known series of tests on tubular specimens, and are later used for computation and comparison with the data of other tests from the same series.

§1. As the author remarks in [1], only the case of a homogeneous stress state is investigated. However, even in this case the description of the material behavior has not been completed. Thus, repeated loadings (i.e., following the process consisting of an active loading and partial or complete unloading) are only examined partially, even the nature of the strain, elastic or plastic, is not mentioned for some loading paths. On the other hand, assumptions are made in that paper about the strong dependence of the moduli g_{ij} on the loadpath (particularly Sec. 1.5, case 3). The strains on the path KL calculated taking account of these assump-

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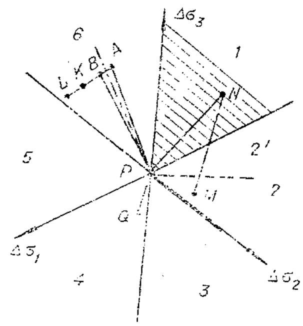

Fig. 1

tions, for example, (Fig. 1 is the deviator plane*), differ by a finite quantity even when the point K is reached by the loadings PBK and PABK (arbitrarily close and coincident on the last portion BK), which is hardly acceptable in the model of a hardening plastic material.

§2. No representation of the loading surface is introduced. The existence of a loading surface has been given a good foundation by experiments and definitions of the loading surface have been taken in traditional plasticity theories: a) as the locus of the elastic limits; b) as the boundaries of the stress state regions obtained because of unloading, where these surfaces coincide. Within the framework of the representations developed in the paper, the mentioned concepts result in different surfaces near the loading point: part of the surface trace a) is shown in the deviator plane on Fig. 1 by heavy lines and is the locus of the elastic limits for active loadings following a total unloading from the state P; the surface b) is the boundary of the domain 1−2'.

§3. The author mentions that some relationships in the paper can be considered a corollary of the Draker−Il'yushin postulate, which is presented in the following formulation; ".... there are no other kinds of internal energy, except elastic energy, in a plastically deformed medium which can be converted into work during deformation." The agreement or connection of the postulate in such a formulation with the Drucker and Il'yushin postulates is not discussed. Application of the relationships given in the paper to some specific loading processes shows that neither the Draker plasticity postulates nor the Il'yushin plasticity postulates are satisfied. Indeed, let the loading occur for unchanged principal axes Σ along the path MNPQPNM (see Fig. 1), the strain process is free (no kinematic constraints, Σ_Δ given); let the path section MNP lie in the initial elastic domain, the state of incomplete plasticity be reached at the point P, PQ be active loading, and QPN be total unloading. Plastic strain then occurs only for the loading PQ, where $\lambda_{13} = 0$, $\lambda_{12} = \lambda_{23} = 1$, $g_{12} = g_{23} = g_0$ ([1], p. 152). Because of the relationships proposed for finding the plastic strains, it is possible to calculate for the path MNPQPNM

$$\int [\sigma_{ij} - \sigma_{ij}(M)]\, de_{ij} = \Delta T_{12}\left[\frac{1}{2g_p}(t_1 - t_3) + \frac{1}{3g_0}(2t_1 - t_2 - t_3)\right] + \Delta T_{23}\left[\frac{1}{2g_p}(t_1 - t_3) + \frac{1}{3g_0}(t_1 + t_2 - 2t_3)\right].$$

Here e_{ij} are the plastic strain tensor components; $\Delta T_{12} = T_{12}(Q) - T_{12}(P)$; $\Delta T_{23} = T_{23}(Q) - T_{23}(P)$; t_1, t_2, t_3 are the principal values of the tensor $\Sigma(P) - \Sigma(M)$; $g_p = g_{13}$. If the points M, P, Q are selected so that $t_1 - t_3 < -2g_p(t_1 + t_2 - 2t_3)/3g_0$ and the ratio $\Delta T_{12}/\Delta T_{23}$ is sufficiently small, then

$$\int [\sigma_{ij} - \sigma_{ij}(M)]\, de_{ij} < 0$$

for the path MNPQPNM, which contradicts the Draker postulate. A closed path in the space of the complete strains ϵ_{ij}, can be indicated analogously, on which negative work $\oint \sigma_{ij} d\epsilon_{ij} < 0$ will be performed, which contradicts the Il'yushin postulate.

§4. The fundamental assumption II is evidently valid for initially isotropic materials in the case of loading processes with unchanging principal axes Σ. However, even for such materials it is unacceptable in a more general loading case. For example, let the specimen be subjected to simple loading, then unloading,

*The state of complete plasticity P is achieved by a simple loading. For loadings which do not alter the principal axes of Σ and are directed into the different domains shown in Fig. 1, the quantities g_{ij}, λ_{ij} are distinct (the loading in the domain 1 is total unloading, in the domain 4, active loading, and in domains 2, 3, 5, 6, partial unloadings). The dashed lines mark the boundaries of domains in which the loading causes partial hardening to set in $T = T_{13}$, $T_1 = T_{12}$, $T_2 = T_{23}$.

and a new loading; let the direction of the principal axes Σ remain unchanged in the new loading, but not coincident with their direction under the first loading. As is known, the fundamental property of a hardening material consists in the preliminary plastic strain exerting influence on the law of the later strain. This influence can be manifested, in particular, in the acquisition of plastic anisotropy (in terms of the plastic strain tensor components), which also results in spoilage of the fundamental assumption II.

§5. A comparison between the test results and computations by means of the scheme proposed in that paper is not sufficiently convincing. This is associated with the following circumstances. For a significant number of quantities selected from test and entering in the computation, the hypotheses, which underlie the whole scheme, have not been verified separately. The selected experiments contain little data about the loading paths with rotation of the principal stress tensor axes: the principal axes Σ in three of the four series of tests were considered to remain fixed throughout the whole loading process and rotation of the principal axes Σ was slight (did not exceed 11.5°) in the last series under the plane stress state conditions, except for specimen B_1 for whose test the rotation of the axes was significant, however, it was accomplished only in the elastic domain during unloading. The discrepancy between computations and the test data of a number of specimens reaches 30% for the strain values at the given loadings (see [1], Figs. 11, 14, 18 (No. 5), 26, 33, etc.).

Therefore, the paper does not contain a final description of a new model of a plastic medium which will permit posing and solving sufficiently general mechanical problems.

LITERATURE CITED

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